Roll No.							

24-ST-21

M.Sc. II SEMESTER [MAIN/ATKT] EXAMINATION
JUNE - JULY 2024

STATISTICS
Paper - I

[Linear Algebra]

[Max. Marks : 75] *[Time : 3:00 Hrs.]* *[Min. Marks : 26]*

Note : Candidate should write his/her Roll Number at the prescribed space on the question paper.
Student should not write anything on question paper.
Attempt five questions. Each question carries an internal choice.
Each question carries **15 marks.**

Q. 1 Define Dimension and basis of a vector space. If V_1 and V_2 are subspace of vectors space then show that

$\dim (V_1 + V_2) \leq \dim (V_1) + \dim (V_2)$

OR

Prove that the necessary and sufficient condition for a vector space V(F) to be the direct sum of its subspace W_1 and W_2 are -

i) $V = W_1 + W_2$

Ii) $W_1 \cap W_2 = \{0\}$

Q. 2 Explain orthogonal projection of a vector. State and prove the necessary and sufficient condition under which a linear transformation is an orthogonal projection.

OR

Define Linear simultaneous equations and solve the following system of linear equation with the help of Cramer's rule -

$-x + 3y + 3z = 6$

$2x + 4y + z = 7$

$3x + 2y + 9z = 14$

Q. 3 What are the various Hermite Canonical Form ? State their properties.

OR

What do you understand by the generalized inverse of a matrix ? Establish its properties.

P.T.O.

Q. 4 Reduce the following quadratic form into canonical form and find its rank, index and signature.

$q = 2x_1^2 + x_2^2 - 3x_3^2 - 8x_2 x_3 - 4x_3 x_1 + 2x_1 x_3$

OR

Which of the following function f defined on vector -

$\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$ in R^2 are bilinear form ?

$f(\alpha, \beta) = x_1 y_2 - x_2 y_1$

Q. 5 Define characteristic matrix, spectrum of a square matrix and characteristic equation. Find the characteristic roots and characteristic vectors of the matrix

$$A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$$

OR

Define eigen values and given vectors of a matrix. Find eigen values and eigen vectors of the following matrix -

$$\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & -1 \\ 2 & -1 & 0 \end{pmatrix}$$

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